(06/21-II)

5259

B. A/B. Sc. EXAMINATION

(For Batch 2011 & Onwards)

(Sixth Semester)

MATHEMATICS

BM-361

Real & Complex Analysis

Time : Three Hours *Maximum Marks :* $\begin{cases} \text{B.Sc.} : 40 \\ \text{B.A.} : 27 \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Unit and the compulsory question. Marks in brackets are for B.A. students.

(Compulsory Question)

1. (a) Show that $u = \sin x + \sin y$, $v = \sin(x+y)$ are not functionally dependent. 1½(1½)

(b) Define Dirichlet's conditions for Fourier expansion. 1½(1½)

(c) Define differentiability of a complex function. 1½(1)

(d) Determine the image of the following points on the sphere of radius $\frac{1}{2}$ and centre $\left(0, 0, \frac{1}{2}\right)$: 1½(1)

(i) $1 + i$

(ii) $1 - i$

(iii) $2 + 3i$

(e) Define conformal mapping. 1(1)

(f) Define the following terms : 1(1)

(i) Magnification

(ii) Rotation.

Unit I

2. (a) Show that the functions $u = x^2 + y^2 + z^2$, and $v = xy - xz - yz$, $w = x + y - z$ are dependent. Also find the relation connecting them. 4(2½)

(b) Prove that :

$$B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

where $m > 0$, $n > 0$. 4(2½)

3. (a) Evaluate : 4(2½)

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz\, dz dy dx$$

(b) Evaluate :

$$\int_0^1 \int_{x^2}^{2-x} xy\, dy dx$$

Change the order of integration and verify the result. 4(2½)

Unit II

4. (a) Obtain the Fourier expansion for $f(x)$,

if $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. 4(2½)

(b) Express $f(x) = x$ as a half range sine series in $0 < x < 2$. 4(2½)

5. (a) Given the series $x = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$,

show that :

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

using Parsevals' identity. 4(2½)

(b) Obtain the Fourier expansion for $f(x)$, if $f(x) = x\sin x$ in $(0, 2\pi)$. 4(2½)

Unit III

6. (a) If the function :

$$f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z - i}$$

continuous at $z = i$? If not, give reason. 4(2½)

(b) Construct the analytic function of which real part is : 4(2½)

$$u(x, y) = e^x (x\cos y - y\sin y)$$

7. (a) Prove that an analytic function with constant modulus is constant. 4(2½)

(b) Show that the function $f(z)=e^{-z^{-4}}\ (z \neq 0)$ and $f(0)=0$ is not analytic at $z = 0$, although C-R equations are satisfied at that point. 4(2½)

Unit IV

8. (a) Let the rectangular region D in the z-plane be bounded by $x = 0$, $y = 0$, $x = 1$, $y = 2$. Determine the region D′ of the *w*-plane *into which* D *is mapped* under the transformation $w=\sqrt{2}e^{i\frac{\pi}{4}}z$. 4(2½)

(b) Prove that every Mobius transformation is the resultant of Mobius transormation with simple geometric imports. 4(2½)

9. (a) Define cross-ratio and show that cross-ratio remains invariant under Mobius transformation. 4(2½)

(b) Find the Mobius transformation which maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ into the points $w_1 = 1$, $w_2 = i$, $w_3 = -1$.