

Roll No.

(011/17-I)

5159**B. A./B. Sc. EXAMINATION**

(First Semester)

MATHEMATICS : ALGEBRA**BM-111**

Time : Three Hours Maxi. Marks : $\begin{cases} \text{B.Sc. : 40} \\ \text{B.A. : 27} \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 1 is compulsory. Marks within Brackets is for B.A. Students.

(Compulsory Question)

1. (a) If A is a square matrix: prove that $A - \bar{A}$ is skew-Hermitian. 2(1)
- (b) Prove that determinant of an orthogonal matrix is ± 1 . 2(1)

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- (c) Find an equation whose roots are four times and with their sign changed of the roots of the equation $4x^4 - x^3 + x + 1 = 0$. 2(1½)
- (d) Show that the equation $2x^7 - 5x^4 + 3x^3 - 1 = 0$ has at least four imaginary roots. 2(1½)

Section I

2. (a) Show that every square matrix can be expressed as the sum of a symmetric and skew-symmetric matrix in one and only one way. 4(2½)
- (b) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ -2 & -4 & 4 & -7 \\ 1 & 2 & 1 & 2 \end{bmatrix} \text{ by reducing it to normal form.} \quad 4(3)$$

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3. (a) Determine the characteristics roots and corresponding characteristic vectors of the

$$\text{matrix } A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}. \quad 4(2\frac{1}{2})$$

- (b) Prove that characteristics roots of a Hermitian matrix are all real. $4(3)$

Section II

4. (a) Solve : $4(2\frac{1}{2})$

$$x + y + 2z + w = 5$$

$$2x + 3y - z - 2w = 2$$

$$4x + 5y + 3z = 7$$

- (b) Show that the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$\text{orthogonal iff } A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \text{ or } \begin{bmatrix} a & b \\ b & -a \end{bmatrix},$$

$$\text{where } a^2 + b^2 = 1. \quad 4(3)$$

5. Reduce the bilinear form $x_1y_1 + x_1y_3 - x_2y_1 + x_2y_2 + x_3y_3$ to the canonical form. Also find the equations of transformation. $8(5\frac{1}{2})$

Section III

6. (a) Solve the equation $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$, roots being in A.P. $4(2\frac{1}{2})$

- (b) Find the common roots of the equations $x^4 + 3x^3 - 5x^2 - 6x - 8 = 0$ and $x^4 + x^3 - 9x^2 + 10x - 8 = 0$. Hence solve them completely. $4(3)$

7. (a) Form the equation whose roots are cubes of the roots of the equation $x^3 + px^2 + qx + r = 0$. $4(2\frac{1}{2})$

- (b) Find the equation of squared differences of the roots of the equation $x^3 - 7x + 6 = 0$. $4(3)$

Section IV

8. (a) Discuss the nature of the roots of the cubic $Z^3 + 3HZ + G = 0$. 4(2½)
- (b) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method. 4(3)
9. (a) Solve the equation $x^4 - 4x^3 + 9x^2 - 12x + 18 = 0$ by Descarte's method. 4(2½)
- (b) Solve the equation $x^4 + 4x^3 + 12x^2 - 8x + 95 = 0$ by Ferrari's method. 4(3)

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