

Roll No. ....

(04/17-I)

**5219**

**B. A./B. Sc. EXAMINATION**

(Fourth Semester)

MATHEMATICS

Sequence and Series

BM-241

*Time : Three Hours    Maximum Marks :*  $\begin{cases} \text{B.Sc.: 40} \\ \text{B.A. : 27} \end{cases}$

**Note :** Attempt *Five* questions in all, selecting at least *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

**Section A**

1. (i) Show that the set  $Z$  is not neighbourhood of any real number.  $Z$  is set of integers.

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P.T.O.

- (ii) Limit pt. of a set may or may not belong to the set. Justify your answer.
- (iii) Define divergent sequence and give example.
- (iv) State D' Alembert's Ratio test for convergence of infinite series of positive terms.
- (v) Prove that if the product  $\prod_{n=1}^{\infty} (1 + a_n)$  is convergent, then  $\lim_{n \rightarrow \infty} a_n = 0$ .
- (vi) Give an example of a series which is convergent but not absolutely convergent.

### Section B

- 2. (a) Prove that between two distinct real numbers, there are infinitely many rational numbers.
- (b) Prove that intersection of a finite number of open sets is an open set.

3. (a) Show that 0 (zero) is the only limit point

of the set  $S = \left\{ \frac{1}{n}, n \in \mathbb{N} \right\}$ .

- (b) Show that a finite set is a compact set.

### Section C

4. (a) Show that a sequence  $\langle a_n \rangle$  is convergent if given  $\epsilon > 0$ , there exist a positive integer  $m$  such that  $|a_n - a_m| < \epsilon$  whenever  $n \geq m$ .

- (b) Show that the sequence  $\langle (-1)^n \rangle$  does not converge.

5. (a) If the series  $\sum_{n=1}^{\infty} a_n$  converges, then show that  $\lim_{n \rightarrow \infty} a_n = 0$ . Is converse true? If not, show by an example.

- (b) Test the convergence of the series :

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots \text{ to } \infty.$$

### Section D

6. (a) State and prove Cauchy's Root Test.  
(b) Discuss the convergence of the series :

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}.$$

7. (a) Test the following series for convergence :

$$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots \quad x > 0.$$

- (b) Discuss the convergence of the series :

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n x^n, \quad x > 0.$$

### Section E

8. State Leibnitz's Test for the convergence of alternating series. And test the convergence of the series :

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots \text{ to } \infty$$

9. Show that the infinite product :

$$\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \dots \text{ is convergent.}$$