Roll No. ....

(09/20-I)

5179

B.A./B.Sc. EXAMINATION

(Second Semester)

MATHEMATICS

(For Re-appear Candidates Only)

BM-121

Number Theory and Trigonometry

Time : Three Hours *Max. Marks* : B.Sc. : 40, B.A. : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Show that g.c.d. $a + b$ and $a - b$ is either 1 or 2 if $(a, b) = 1$. 2(2)

P.T.O.

(b) Show that $2^4 / \phi(1155)$. 2(1)

(c) Find all the values of $(1 + i)^{1/3}$ and obtain their product. 2(2)

(d) Resolve into real and imaginary parts $\log(4+3i)$. 2(2)

Section I

2. (a) Show that there are infinite many primes of the form $4n + 3$. 4(2½)

(b) Solve the congruence $15x \equiv 12 (\mathrm{mod}\, 21)$. 4(2½)

3. (a) Find all the solutions in positive integers of $5x + 3y = 52$. 4(2½)

(b) If $(P - 1)! + 1 = 0 (\mathrm{mod}\ P)$, then prove that P is a prime number. 4(2½)

Section II

4. (a) Prove that $\phi(n) = \phi(n + 2)$ is satisfied by $n = 2(2p - 1)$, wherever p and $(2p - 1)$ are both odd prime. 4(2½)

(b) Find the highest power of 180 in 102 ! 4(2½)

5. (a) Find positive integer n which satisfy $\mu(n) + \mu(n+1) + \mu(n+2) = 3$. 4(2½)

(b) Find value of $\left(\frac{a}{11}\right)$ for $a = 3, 7$. 4(2½)

Section III

6. (a) If α, β are roots of $x^2 - 2x + 4 = 0$. Prove that $\alpha^n + \beta^n = 2^{n+1}.\cos\frac{n\pi}{3}$. 4(2½)

(b) Show that :

$$\tan\frac{\theta}{7} + \tan\frac{\theta+\pi}{7} + \ldots \tan\frac{\theta+6\pi}{7} = 7\tan\theta$$

using De Moivre theorem. 4(2½)

7. (a) If $Z = x + iy$, where x and y are real, find real and imaginary parts of $\frac{\cos Z}{Z+1}$. 4(2½)

(b) If $\cos(\alpha - i\beta) = 1$, show that :

$$\sin^2\alpha = \sinh^2\beta$$ 4(2½)

Section IV

8\. (a) Express $\log \sin(x+iy)$ in the form of A + iB. 4(2½)

(b) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, show that $x^2 + y^2 + z^2 + 2xzy = 1$. 4(2½)

9\. (a) Prove that : 4(2½)

$$\log \tan\left(\frac{\pi}{4} + \frac{x}{2} i\right) = i \tan^{-1}(\sinh x)$$

(b) Sum the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right)$.......... to n terms and deduce the sum to infinity. 4(2½)