Roll No. ....

(09/20-I)

5181

B.A./B.Sc. EXAMINATION

(Second Semester)

MATHEMATICS

(For Re-appear Candidates Only)

BM-123

Vector Calculus

Time : Three Hours *Max. Marks :* $\begin{cases} \text{B.Sc.} : 40 \\ \text{B.A.} : 27 \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) What is the greatest rate of increase of $u = xyz^2$ at the point (1, 0, 3) ? 1½(1)

(b) Show that $\dfrac{d\phi}{ds} = \nabla\phi . \dfrac{d\vec{r}}{ds}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and ϕ is a function of x, y, z. 1½(1)

(c) Find the value of λ, so that the following vectors : $\vec{a} = 2\hat{i} - j + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar. 1½(1)

(d) Write the formula's of $\nabla\phi$ and $\nabla^2\phi$ in terms of orthogonal curvilinear co-ordinates u, v and w. 1½(1)

(e) Evalaute $\oint_C \vec{r}.d\vec{r}$, where C is a circle repersented by $x^2 + y^2 = a^2, z = 0$. 2(1)

Section I

2. (a) If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{1}{2}\vec{b}$. Find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$, where $\vec{b}$ and $\vec{c}$ are non-parallel. 4(3)

(b) A particle moves along the curve $x = 3t^2$, $y = t^2 - 2t$, $z = t^3$. Find its velocity and acceleration at $t = 1$ in the direction of vector $\vec{a} = \hat{i} + \hat{j} - \hat{k}$. 4(2½)

3. (a) Prove that the necessary and sufficient condition for the vector function $\vec{f}$ of a scalar variable t to have a constant direction is $\vec{f} \times \dfrac{d\vec{f}}{dt} = \vec{0}$. 4(3)

(b) Given $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 2i + \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$. Prove that : 4(2½)

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]\left[\vec{a}'\ \vec{b}'\ \vec{c}'\right] = 1.$$

Section II

4. (a) Find the constants a and b so that the surface $ax^2 - byz - (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, −1, 2). 4(3)

(b) Show that : 4(2½)

$$\nabla^2\left[\nabla.\frac{\vec{r}}{r^2}\right] = 2r^{-4}.$$

5. (a) Evaluate $\nabla.\left(\frac{\vec{r}}{r}\right)$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = r$. 4(3)

(b) Prove that : 4(2½)

$$\frac{2}{r}f'(r) + f''(r) = \nabla^2 f(r).$$

Section III

6. (a) Represent the vector $\vec{A} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical co-ordinates. Hence determine A_p, A_ϕ and A_z. 4(3)

(b) Prove that $\frac{d}{dt}(\hat{e}_p) = \dot{\phi}\hat{e}_\phi$ and $\frac{d}{dt}(\hat{e}_\phi) = -\dot{\phi}\hat{e}_\rho$, where dots denotes the differentiation w.r.t. time t. 4(2½)

7. (a) Show that in any orthogonal curvilinear system, div (curl $\vec{A}$) = 0 and curl(grad ϕ) = $\vec{0}$. 4(3)

(b) Express the velocity and acceleration of a particle in spherical co-ordinates. 4(2½)

Section IV

8. (a) Find the circulation of $\vec{f}$ round the curve C, where $\vec{f} = y\hat{i} + z\hat{j} + x\hat{k}$ and C is the circle $x^2 + y^2 = 1$, $z = 0$. 4(3)

(b) Evaluate $\iint_S \vec{f}.\hat{n}dS$, where $\vec{f} = z\hat{i} + x\hat{j} + 3y^2 z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. 4(2½)

9. (a) Evaluate $\iint_S (\nabla \times \vec{f}).\hat{n}ds$, where S is the surface of the cone $z = 2 - \sqrt{x^2 + y^2}$ above xy-plane. and $\vec{f} = (x - z)\hat{i} + (x^3 + yz)\hat{j} - 3xy^2\hat{k}$. 4(3)

(b) Verify Green theorem in the xy-plane for $\oint_C \left(xy^2 - 2xy\right)dx + \left(x^2 y + 3\right)dy$ around the boundary C of the region inclosed by $y^2 = 8x$ and $x = 2$. 4(2½)