Unit III

6. (a) Expand $x^2y+3y-2$ in powers of $(x-1)$ and $y+2$. 7

(b) If V is a function of two variables x and y and $x=r\cos\theta$, $y=r\sin\theta$, prove that :

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=\frac{\partial^2 V}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2}+\frac{1}{r}\frac{\partial V}{\partial r}$$ 7

7. Show that :

$$f(x, y, z)=(x+y+z)^3-3(x+y+z)-24xyz+a^3$$

has a minima at (1, 1, 1) and a maxima at (–1, –1, –1). 14

Unit IV

8. State and prove Abel's theorem (First form and 2nd form) in power series. 14

9. State and prove Taylor's theorem in power series. 14

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Roll No. ....

(12/19-II)

11642

M. Sc. (2 Year) EXAMINATION

(For Batch 2017 & Onwards)

(First Semester)

MATHEMATICS

MTHCC-2102

Real Analysis

Time : Three Hours *Maximum Marks* : 70

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory consisting of seven questions of 2 marks each. Further the paper is divided into four Units each consisting of two questions. Candidate is required to attempt *one* question from each Unit consisting of 14 marks for each. Marks are indicated with question.

P.T.O.

1. (a) If $f \in \mathbf{R}(\alpha)$ and $g \in \mathbf{R}(\alpha)$ on $[a, b]$, then $fg \in \mathbf{R}(\alpha)$.

(b) Prove that :

$$\int_0^3 x\left(d[x]-x\right)=\frac{3}{2}$$

(c) Show that $\frac{x}{1+nx^2}$ is uniformly convergent in R.

(d) State and prove Cauchy's general principle of uniform convergence for sequences.

(e) Show that $f(x, y)=\sqrt{|xy|}$ is not differentiable at the point (0, 0).

(f) If $u=\phi(x+at)+\psi(x-at)$, show that :

$$\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$$

(g) Define orthogonal system of functions in a Fourier series. 2×7=14

Unit I

2. (a) Define R-S integral. Show the existence of the Riemann-Stieltjes integral. 7

(b) If lim S(P, f, α) exists as $\mu(P) \to 0$, then $f \in \mathbf{R}(\alpha)$, and

$$\lim_{\mu(P)\to 0} S(P, f, \alpha)=\int_a^b f d\alpha.$$ 7

3. (a) State and prove first mean value and 2nd mean value theorem. 7

(b) If $f \in \mathbf{R}(\alpha)$ on $[a, b]$ and if $a < c < b$, then $f \in \mathbf{R}(\alpha)$ on $[a, c]$ and on $[c, b]$, and : 7

$$\int_a^b f d\alpha=\int_a^c f d\alpha+\int_c^b f d\alpha$$

Unit II

4. State and prove Dini's theorem. 14

5. State and prove Weierstrass approximation theorem. 14