Roll No. ....

(01/21-II)

11642

M.Sc. (2 Years) EXAMINATION

(For Batch 2017 & Onwards)

(First Semester)

MATHEMATICS

MTHCC-2102

Real Analysis

Time : Three Hours *Maximum Marks* : 70

Note : Attempt *Five* questions in all including compulsory question and *one* question from each Unit.

(Compulsory Question)

1. (a) Assume that α is increasing on $[a, b]$, then $\underline{I}(f, \alpha) \leq \bar{I}(f, \alpha)$.

(b) State first and second mean value theorem for Riemann-Stieltjes integrals.

(c) Let $f_n(x) = \frac{\sin nx}{\sqrt{n}}$, ($x$ real), show that the limit of differentials of $f_n(x)$ may not equal to the differential of the limit.

(d) State and prove Weierstrass M-test for uniform convergence of series.

(e) Define Chain Rule and directional derivative.

(f) State inverse function theorem.

(g) Find the radius of convergence of $\sum_{n=0}^{\infty} z^n$ and $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$. Which one is convergent everywhere and which are convergent nowhere on the boundary of the disk of convergence.

Unit I

2\. (a) State and prove the formula for integration by parts in a Riemann-Stieltjes integral.

(b) State and prove the theorem which permits to replace $d\alpha(x)$ by $\alpha'(x)dx$ in $\int_a^b f(x)d\alpha(x)$. Is there any condition on α in $[a, b]$.

3\. (a) Assume that α is increasing on $[a, b]$. If $f \in R(\alpha)$ on $[a, b]$, then prove that f satisfies Riemann's condition w.r.t. α on $[a, b]$.

(b) Prove that the function given by $f(x) = x\cos\left(\frac{\pi}{2x}\right)$ for $x \neq 0$, $f(0) = 0$, is not rectifiable curve.