Roll No. ....

(06/21-II)

11671

M. Sc. (2 Years) EXAMINATION

(For Batch 2017 & Onwards)

(Fourth Semester)

MATHEMATICS

MTHCC-2401

Functional Analysis

Time : Three Hours *Maximum Marks* : 70

Note : Question No. **1** is compulsory. Attempt *Five* questions in all, selecting *one* question from each Unit including compulsory question. All questions carry equal marks.

(Compulsory Question)

1. (a) If the metric d defined by $d(x,y)=\|x-y\|$, where $x,y \in N$. Show that normal linear space N is a metric space w.r.t. distance function d defined above. 2

(b) Define a second conjugate space. 2

(c) Show that strong convergence implies weak convergence but converse is not true. 2

(d) Write down conjugate space of l_1. 2

(e) Let X and Y be normed linear spaces and let D be a closed subspace of X. If $T : D \to Y$ is bounded, then T is closed. 2

(f) If $\langle e_i \rangle$ is an orthonormal set in a Hilbert space H and if x is any vector in H, then the set $S = \{e_i : (x, e_i) \neq 0\}$ is either empty or countable. 2

(g) An operator on a Hilbert space H is normal if and only if $\|T^* x\| = \|Tx\|$ for every x. 2

Unit I

2. (a) Let N be a normal linear space over the scaler field F. Then : 7

(i) The map $(\alpha, n) \to \alpha x$ from F × N $\to$ N is continuous.

(ii) The map $(x, y) \to x + y$ from N × N $\to$ N is continuous.

(iii) The map $x \to \|x\|$ from N to R is continuous.

(b) State and prove F-Ritz's Lemma. 7

3. Under usual notations show that $B(N, N')$ is a complete normed linear space. 14

Unit II

4. (a) State and prove uniform boundedness principle. 7

(b) Show that $C[0,1]$ is not reflexive. 7

5. Let M be a closed linear subspace of a normed linear space N and let x_0 be a vector not in M. If d is the distance from x_0 to M, show that there exists a functional $f_0 \in N^*$ s.t. : $(f_0(M) = \{0\},\ f_0(x_0) = d \text{ and } \|f_0\| = 1)$. 14

Unit III

6. State and prove open mapping theorem. 14

7. (a) If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$, then the linear subspace $M + N$ is also closed. 7

   (b) State and prove Cauchy-Schwarz inequality. 7

Unit IV

8. (a) Let H be a Hilbert space and let $\langle e_i \rangle$ be an orthonormal set in H. Then the following conditions are all equivalent to one another. 7

   (b) The real Banach space of all self-adjust operators on H is a partially ordered set whose linear structure and order structure are related by the following properties : 7

   (i) If $A_1 \leq A_2$, then $A_1 + A \leq A_2 + A$ for every A.

   (ii) $A_1 \leq A_2$ and $\alpha \geq 0$, then $\alpha A_1 \leq \alpha A_2$.

9. (a) If A is a political operator on H, then $I + A$ is nonsingular. In particular $I + T^*T$ and $I + TT^*$ are non-singular for an arbitrary operator T on H. 7

   (b) If T is an operator on H, then T is normal $\Leftrightarrow$ its real and imaginary parts commute. 7