Roll No. ....

(07/21-II)

11659

M. Sc. (2 Year) EXAMINATION

(For Batch 2019 & Onwards)

(Second Semester)

MATHEMATICS

MTHCC-2202

Measure & Integration Theory

Time : Three Hours *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) If $M^*(A) = 0$, then prove that $M^*(A \cup B) = M^*(B)$.

   (b) Define measurable set and prove that difference of two measurable set is measurable.

(c) Prove that characteristic function of a set A is measurable if A is measurable.

(d) Show that every function defined on a set of measure zero is measurable.

(e) Give an example to show that the Lebesgue integral of a no where zero function can be zero.

(f) Prove that in Lebesgue dominated convergence theorem, the existence of dominant function is sufficient condition, not necessary.

(g) Prove that every function of bounded variation is bounded.

Unit I

2. (a) Prove that outer measure of countable set is zero. Is the converse true if not provide suitable example ?

(b) Show that a set is Lebesgue measurable if and only if its complement is measurable.

3. (a) If E_1, E_2, E_3, ...are pairwise disjoint measurable sets and $E = E_1 \cup E_2 \cup E_3 \cup$ ...., then show that E is measurable and $m(E) = \sum_{r=1}^{\infty} m(E_r)$.

(b) Prove that the set of type F_σ and G_δ are measurable sets.

Unit II

4. (a) Show that a function is simple if and only if it is measurable and assumes a finite number of values.

(b) State and prove Lusin's theorem.

5. (a) Let f be a function defined on measurable set E. Then f is measurable iff for any open set O in R, $f^{-1}(O)$ is measurable set.

(b) State and prove F. Riesz's theorem.

Unit III

6. (a) Show that the function f defined on the interval $[a, b]$ by :

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$$

is Lebesgue integrable, but not Riemann integrable.

(b) State and prove Fatou's Lemma.

7. (a) Verify the result of bounded convergence theorem for the function :

$$f_n(x) = \frac{nx}{1 + n^2 . x^2}, 0 \le x \le 1$$

(b) State and prove necessary and sufficient condition of Lebesgue integrability.

Unit IV

8. (a) State and prove Jordan de-composition theorem.

(b) Show that the function f defined on [0, 1] by :

$$f(x) = \begin{cases} x.\sin\frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

is not of bounded variation.

9. State and prove Lebesgue differentiation theorem.