Roll No. ....

(07/21-II)

11658

M. Sc. (2 Years) EXAMINATION

(For Batch 2019 & Onwards)

(Second Semester)

MATHEMATICS

MTHCC-2201

Advanced Abstract Algebra

Time : Three Hours *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit including compulsory Q. No. 1. All questions carry equal marks.

(Compulsory Question)

1. (a) Prove that characteristic of a field is equal to the characteristic of its sub-fields. 2

(b) Define algebraically closed field. Show that algebraically closed fields are infinite always. 2

(c) Show that $[K:Q]=2$. 2

(d) If F is a finite field of characteristics p then $a \to a^p$ is an automorphism of F. 2

(e) A square of the area equal to area of the unit circle is not constructible by ruler and compass. 2

(f) If T is nilpotent, then I – T is regular (or invertible). 2

(g) Under usual notations define $C(f(x))$. 2

Unit I

2. (a) Any prime field is either isomorphic to the field of rational numbers or to the field of integers modulo some prime number. 7

(b) If L is an algebraic extension of K and K is an algebraic extension of F, then L is an algebraic extension of F. 7

3. (a) If $f(x)$ is any polynomial of degree $n \geq 1$ over a field F, then there exists a field extension E of F such that $f(x)$ has n roots in E and $[E:F] \leq n!$. 7

(b) Find the splitting and degree of the polynomial $x^5 - 1$. 7

Unit II

4. (a) If an element $a \in K$ is algebraic over F, then there exists a unique monic polynomial $p(x)$ of positive degree over F, such that (i) $p(a)=0$ (ii) if any $f(x) \in F[x]$, $f(a)=0$, then $p(x)$ divides $f(x)$. 7

(b) Let $p(x)$ be an irreducible polynomial in $F[x]$ and $p'(t)$, then corresponding polynomial in $F'(t)$. Suppose u and v are roots of $p(x)$ and $p'(t)$ respectively in some field extension E and E′ of F and F' respectively, then there exists an isomorphism μ of $F(u)$ onto $F'(v)$ such that $\mu(\alpha) = \alpha' \ \forall \alpha \in F$ and $\mu(u) = v$. 7

5. (a) An irreducible polynomial $f(x)$ over a field F of characteristic $p > 0$ is inseparable if and only if $f(x) \in F\left[x^p\right]$, i.e. $f(x)$ is a polynomial in x^p. 7

(b) Let K be a finite algebraic extension a field F. Then K is a normal extension of F if and only if K is the splitting field over F of some non-zero polynomial over F. 7

Unit III

6. (a) The set of all automorphisms of a field form a group under resultant composition. 7

(b) Show that $\phi_n(x) \in Z[x]$. 7

7. State and prove Fundamental theorem of Galois theory. 14

Unit IV

8. (a) If $T \in A(V)$ $(\dim V = n)$ has all its characteristic roots in F, then there is a basis of V in which the matrix of T is triangular. 7

(b) If $u \in V_1$ is such that $uT^{n_1 - k} = 0$, where $0 < k < n_1$, then $u = 40T^k$ for some $u_0 \in V_1$. 7

9. The elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors. 14