Roll No. ....

(07/21-II)

11651

M. Sc. (2 Years) EXAMINATION

(For Batch 2017 to 2018 Only)

(Second Semester)

MATHEMATICS

MTHCC-2201

Advanced Abstract Algebra

Time : Three Hours *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **9** is compulsory. All questions carry equal marks.

Unit I

1. (a) If K/F be any extension and let S = $\{x \in K : x$ is algebraic over F$\}$. Then S is a subfield of K containing F and S is the largest algebraic extension of F contained in K. 7

(b) Let $\alpha \in K/F$ be algebraic of odd degree. Prove that $F(\alpha) = F(\alpha^2)$. 7

2. (a) Find the splitting field and its degree for the polynomial $x^4 - 1$ over Q. 7

(b) Let F be a field. Then there exists an algebraically closed field containing F as a sub-field. 7

Unit II

3. (a) For every prime p and integer $n \geq 1$, there exists a field having p^n elements. 7

(b) Let F be a field of characteristic $p \neq 0$ and K/F be any extension. An element $a \in K$ is separable ove F iff F(a) is a separable extension of F. 7

4. (a) For every positive integer n the polynomial $\phi_n(x)$ is irreducible over the field of rational numbers. 7

(b) Let K be a finite algebraic extension of a field F then K is a normal extension of F iff K is the splitting field of some non-zero polynomial over F. 7

Unit III

5. (a) Let K be a finite extension of finite field of F. Let K/F is a Galois extension. Then $G(K/F)$ is a cyclic group. 7

(b) Find the Galois group of $x^4 - 8x^2 + 15$ over Q. 7

6. (a) A regular n-gon is constructive if and only if $\phi(n)$ (Euler function) is a power of 2. 7

(b) State and prove Fundamental theorem of Algebra. 7

Unit IV

7. (a) Let W be a subspace of V and $T \in A(V)$. Suppose $WT \subseteq W$. Then :

(i) There exists a linear transformation

$$\overline{T} : \frac{V}{W} \to \frac{V}{W}$$

(ii) If $p(x) \in F[x]$ is the minimal polynomial of T and $p_1(x) \in F[x]$ is the minimal polynomial of $\overline{T}$, then $p_1(x)$ divides $p(x)$. 7

(b) Let $T \in A(V)$ and $V = V_1 \oplus V_2$, where $V_1 T \subseteq V_1$ and $V_2 T \subseteq V_2$, then $\exists$ a basis of V such that the matrix of T in this basis is of the form $\begin{bmatrix} A & O \\ O & B \end{bmatrix}$ and A is the matrix of T on V_1 and B is the matrix of T on V_2. 7

8. State and prove Primary decomposition theorem. 14

(Compulsory Question)

9. (a) Define splitting field with example.

(b) Let K/F be any extension and $f(x) \in F(x)$ then an element 'a' $\in$ K is a root of the polynomial $f(x)$ iff $(x-a)/f(x)$ in $K[x]$.

(c) A field F is called perfect if all its finite extensions over separable.

(d) Prove that if characteristics F = 0, then any algebraic extension of F is always separable.

(e) Define Cyclic extension with example.

(f) State and explain Jordan Canonical form.

(g) State Galois extension. $2 \times 7 = 14$