Roll No. ....

(07/21-II)

9191

M. Sc. (2 Years) EXAMINATION

(For Batch 2013 to 2016 Only)

(Second Semester)

MATHEMATICS

MMT-4201

Advanced Abstract Algebra–II

Time : Three Hours *Maximum Marks* : 80

Note : Attempt all parts of Section A, *five* questions from Section B and *two* questions from Section C.

Section A

1. (a) Prove that the relation of similarity is an equivalence relation in A(V). 2

   (b) Write the companion matrix for $(1+x)^n$. 2

Section C

10. There exists a sub-space W of V, invarient under T s.t. $V = V_1 \oplus W$. 12

11. Two nilpotent linear transformations are similar if and only if they have the same invarients. 12

12. State and prove Hilbert basis theorem. 12

13. State and prove Wedderburn-Artin theorem. 12

(c) Define index of nilpotency. 2

(d) Show that Q is not a free Z-module. 2

(e) Define a semi-simple module. 2

(f) Find the abelian group generated by (x_1, x_2), subject to $2x_1 = 0$, $3x_2 = 0$. 2

(g) Define a primary module. 2

(h) Show that Z is not artinian. 2

Section B

2. If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then T satisfies a polynomial of degree n over F. 8

3. If $u \in V_1$ is st $uT^{n_1-k} = 0$, where $0 < k < n_1$, then $u = 40T^k$ for some $u_0 \in V_1$. 8

4. Let V be a Vector space and $T \in A(V)$. Suppose :

$$p(x) = a_0 + a_1x + ... + a_{m-1}x^{m-1} + x^m \in F[x] ,$$

be the minimal polynomial of T. Also suppose

that V is cyclic $F[x]$-module, then $\exists$ a basis of V s.t. the matrix of T in this basis is $C(p(x))$. 8

5. State and prove Schur's Lemma. 8

6. Let M be a free R-module with basis $\{x_1, x_2, ..., x_n\}$, then $R^n \cong M$. 8

7. Let $M_1, M_2, ..., M_k$ be Noetherian submodules, then $\sum_{i=1}^{k} M_i$ is also Noetherian. 8

8. Let R be a commutative ring with unity and I be the finitely generated ideal of R s.t. $I = I^2$. Then I is the direct summand of R. 8

9. Find the abelian group generated by $\{x_1, x_2, x_3\}$,

subject to : $5x_1 + 9x_2 + 5x_3 = 0$

$2x_1 + 4x_2 + 2x_3 = 0$

$x_1 + x_2 - 3x_3 = 0$. 8