

Roll No.

(05/16-I)

5219

B.A./B.Sc. EXAMINATION

(Fourth Semester)

MATHEMATICS

Sequences & Series

BM-241

Time : Three Hours Maximum Marks : $\begin{cases} \text{B.Sc. : 40} \\ \text{B.A. : 27} \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory. Marks in bracket are shown for B.A.

1. (a) Define limit point of a set. 1(1)
(b) Discuss the boundedness of the sequence :

$$\left\langle \frac{2n+3}{3n+4} \right\rangle \quad 1(1\frac{1}{2})$$

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P.T.O.

- (c) State Cauchy's Principle of Convergence.

2(1½)

- (d) Show that every absolutely convergent series is convergent.

2(1½)

- (e) Examine the convergence of the product :

$$\prod_{n=1}^{\infty} \cos \frac{\theta}{n} \quad 2(1½)$$

Section I

2. (a) Prove that every non-empty set of real numbers which is bounded below has g.l.b.

4(2½)

- (b) Show that the set of real is not a compact set.

4(2½)

3. (a) Prove that the intersection of an arbitrary family of closed sets is closed.

4(2½)

- (b) Prove that the set :

4(2½)

$$\left\{ -1, 1, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{1}{3}, \dots \right\}$$

is neither open nor closed.

Section II

4. (a) Prove that if a sequence $\langle a_n \rangle$ converges to a and $b_n = \frac{a_1 + a_2 + \dots + a_n}{n}$, then the sequence $\langle b_n \rangle$ also converges to a .

4(2½)

- (b) Show that the sequence $\langle a_n \rangle$ defined by

$$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2n+1} \quad \text{does not converge.}$$

4(2½)

5. (a) Show that $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{6^n}$ converges.

4(2½)

- (b) Prove that a necessary and sufficient condition for the convergence of a

positive term series $\sum_{n=1}^{\infty} a_n$ is that the sequence $\langle S_n \rangle$ of the partial sums of the series defined by $S_n = a_1 + a_2 + \dots + a_n$ is bounded above.

4(2½)

Section III

6. (a) Test the convergence of the series :

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{x}{4 \cdot 5 \cdot 6} + \frac{x^2}{7 \cdot 8 \cdot 9} + \dots (x > 0) \quad 4(2\frac{1}{2})$$

- (b) State and prove Cauchy's Root Test. 4(2\frac{1}{2})

7. (a) Using Cauchy's condensation test, discuss the convergence of the series : 4(2\frac{1}{2})

$$\sum_{n=1}^{\infty} \frac{1}{n(\log n)^p}$$

- (b) Discuss the convergence of the series :

$$\frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \dots \quad 4(2\frac{1}{2})$$

Section IV

8. (a) Test the convergence of the following series : 4(2\frac{1}{2})

$$\frac{1}{\log 2} - \frac{1}{\log 3} + \frac{1}{\log 4} - \frac{1}{\log 5} + \dots$$

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- (b) Show that the Cauchy's product of the convergent series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ with itself is divergent. 4(2\frac{1}{2})

9. (a) Prove that the series $\sum_{n=1}^{\infty} |\log(1+a_n)|$ is convergent if and only if $\sum_{n=1}^{\infty} |a_n|$ is convergent. 4(2\frac{1}{2})

- (b) Show that the series :

$$\left(1 - \frac{1}{2}\right) + \left(1 - \frac{3}{4}\right) + \left(1 - \frac{7}{8}\right) + \dots$$

is convergent but when parenthesis are removed, it oscillates. 4(2\frac{1}{2})

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