Roll No. ....

(07/20-I)

5260

B. A./B. Sc. EXAMINATION

(Sixth Semester)

MATHEMATICS

BM-362

Linear Algebra

Time : Three Hours *Maximum Marks :* B. Sc. : 40, B. A. : 26

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. Marks outside the brackets are for B.Sc. students and marks within the brackets are for B.A. students.

(Compulsory Question)

1. (a) Define a Vector Space. 2(1½)

(b) Show that the set $\{1, i\}$ generates C (set of complex numbers) over R (set of real numbers). 2(1½)

(c) Define singular and non-singular linear transformation. 2(1½)

(d) Define orthogonal set in an inner product space. 2(1½)

Unit I

2. (a) Prove that union of two subspaces of a vector space, is a subspace if and only if one is contained in the other. 4(2½)

(b) Show that the set $\{(1, 2, 3), (0, 1, 2), (0, 0, 1)\}$ generates the vector space $R^3(R)$. 4(2½)

3. (a) If V(F) is a finitely generated vector space, then prove that any maximal linearly independent subset of V(F) is a basis of V(F). 4(2½)

(b) If W is a subspace of a vector space V(F) and $u, u' \in V$, then W + u = W + u' iff $u - u' \in W$. 4(2½)

Unit II

4. (a) Show that linear transformation $T : R^2 \to R^2$ defined by :

$$T(x_1, x_2) = (x_1 \cos\theta + x_2 \sin\theta, \; - x_1 \sin\theta + x_2 \cos\theta)$$

is a vector space isomorphism. 4(2½)

(b) Let $T : U \to V$ be a linear transformation, then U/ker T $\cong$ T(U). 4(2½)

5. (a) If $T : U \to V$ is a linear transformation, then Rank T + Nullity T = Dim. U. 4(2½)

(b) Let $S = \{v_1, v_2, v_3\}$ be a basis of $V_3(R)$, defined by $v_1 = (-1, 1, 1)$, $v_2 = \{1, -1, 1\}$, $v_3 = \{1, 1, -1\}$. Find the dual basis of S. 4(2½)

Unit III

6. (a) Let $T : R^3 \to R^3$ be a linear operator defined by $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Show that T is invertible and find T^{-1}. **4(2½)**

   (b) If C(R) is a vector space and T is a linear transformation on C(R) defined by $T(a + ib) = a - ib$ for all $a, b \in R$; find the matrix of T with respect to ordered basis $B = \{1 + i, 1 + 2i\}$. **4(2½)**

7. (a) Let U and V be finite dimensional vector spaces and let $B = \{u_1, u_2, ..., u_n\}$ and $B = \{v_1, v_2, ..., v_n\}$ be ordered basis for U and V respectively. If T is a linear transformation from U to V then for any $u \in U$:

$$[T(u), B'] = [T : B, B'] [u, B].$$

4(2½)

   (b) Show that the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable over the field C. **4(2½)**

Unit IV

8. (a) State and prove Cauchy Schwarz inequality. **4(2½)**

   (b) Prove that every finite dimensional vector space is an inner product space. **4(2½)**

9. (a) Obtain an orthonormal basis with respect to standard inner product for subspace of R^3 generated by (1, 0, 1), (1, 0, – 1) and (0, 3, 4). **4(2½)**

   (b) Let T be a linear operator on a finite dimensional inner product space V(F). Show that T can be uniquely expressed as the sum of a self-adjoint and skew-symmetric operator. **4(2½)**