Roll No. .....

(12/19-II)

5161

B. A./B. Sc. EXAMINATION

(First Semester)

MATHEMATICS

BM-113

Solid Geometry

Time : Three Hours *Maximum Marks :* B. Sc. : 40, B. A. : 27

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Find the nature of the curve :

$$2x^2 - 72xy + 23y^2 - 4x - 28y - 48 = 0 \ldots$$

(b) Find the equation of the sphere on join of points (–1, 3, 2) and (5, 7, – 6) as diameter end points.

(c) Prove that one conicoid confocal with a given conicoid, touch a plane.

(d) If a right circular cone has three mutually perpendicular tangent planes, then show that the semi-vertical angle is $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

(e) Find the equation of the plane which cuts the paraboloid $x^2 - 2y^2 = z$ in a conic with its centre at the point $\left(2, \frac{3}{2}, 4\right)$.

Unit I

2. Find the length of the axes, the eccentricity and the equations of the axes of the conic $5x^2 + 6xy + 5y^2 + 4x + 12y - 4 = 0$.

3. (a) Find the condition that the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ may be tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$.

(b) If two confocal ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ are cut by a straight line $x\cos\alpha + y\sin\alpha = p$ and if T, T' be the poles w.r.t. confocal ellipse of the straight line, then prove that TT' = $\frac{\lambda}{p}$.

Unit II

4. (a) A sphere of radius K, passes through the origin and meets the axes in A, B and C. Prove that the centroid of the triangle ABC lies on the sphere :

$$9(x^2 + y^2 + z^2) = 4k^2.$$

(b) Find the limiting points of the co-axial system determined by the two spheres whose equation are :

$$x^2 + y^2 + z^2 - 8x + 2y - 2z + 32 = 0$$

and $x^2 + y^2 + z^2 - 7x + z + 23 = 0$.

5. (a) Prove that the equation of the right circular cone whose vertex is the origin, axis the y-axis and semivertical angle α is $x^2 + z^2 = y^2 \tan^2 \alpha$.

(b) Find the equation of the right circular cylinder of radius 3 and axis as the line

$$\frac{1-x}{-2} = \frac{y}{2} = \frac{3-z}{-1}.$$

Unit III

6. (a) Prove that the sum of the squares of the reciprocals of any *three* mutually perpendicular diameters of an ellipsoid is constant.

(b) Prove that the six normals from a point to an ellipsoid lie on a curve of second degree.

7. (a) Show that the centre of the conic $lx + my + nz = p$, $ax^2 + by^2 + cz^2 = 1$ is the point :

$$\left(\frac{lp}{ap_0^2}, \frac{mp}{bp_0^2}, \frac{np}{cp_0^2} \right)$$

where

$$p_0^2 = \frac{l^2}{a} + \frac{m^2}{b} + \frac{n^2}{c}.$$

(b) Show that the plane $2x - 4y - z + 3 = 0$ touches the paraboloid $x^2 - 2y^2 = 3z$. Also find the point of contact.

Unit IV

8. (a) Find the co-ordinates of the centre and the lengths of the semi-axes of the section of ellipsoid $3x^2 + 3y^2 + 6z^2 = 10$ by the plane $x + y + z = 1$.

(b) Find the equations of the generating lines of the hyperboloid :

$$yz + 2zx + 3xy + 6 = 0$$

which pass through the point $(-1, 0, 3)$.

9. (a) If a straight line touches two confocal conicoids, prove that the tangent planes at the point of contact are at right angles.

(b) Reduce the equation :

$$9x^2 + 4y^2 + 4z^2 + 8yz + 12zx + 12xy + 4x + y + 10z + 1 = 0$$

to the standard form. Show that it represents a parabolic cylinder.