Roll No. ....

(04/17-I)

5260

B.A./B.Sc. EXAMINATION

(Sixth Semester)

MATHEMATICS

BM-362

Linear Algebra

Time : Three Hours *Maximum Marks :* $\begin{cases} \text{B.Sc.} : 40 \\ \text{B.A.} : 26 \end{cases}$

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. Q. No. **1** is compulsory. Marks in brackets are meant for B.Sc.

Compulsory Question

1. (a) Show that the map $T : R^3 \rightarrow R^2$ defined by $T(x, y, z) = (|x|, y - z)$ is not a linear transformation. 1(2)

(b) Define rank and nullity of a linear transformation. 1(2)

(c) Express $V = (1, -2, 5)$ as a linear combination of the vectors $V_1 = (1, 1, 1)$, $V_2 = (1, 2, 3)$, $V_3 = (2, -1, 1)$ in the vector space $R^3(R)$. 1(2)

(d) Find the coordinates of the vector $(-1, 2, 3, 4)$ relative to ordered basis $B = \{(0, 1, 1, 0), (0, 0, 1, 1), (1, 0, 0, 1), (0, 1, 0, 0)\}$ for V_4. 1(2)

(e) The union of two subspaces of a vector space V(F) may not be a subspace of V(F). Give example. 1(2)

Unit I

2. (a) If W is a subspace of a finite dimensional vector space V(F), then W is finite dimensional and dim W $\leq$ dim V. Also dim W = dim V iff W = V. 3(4)

(b) Determine a basis of the subspace spanned by the vectors (3, 2, 4), (1, 0, 2), (1, −1, −1) and (6, 7, 5). 3(4)

3\. (a) If W_1 and W_2 are two subspaces of a finite dimensional vector space $V(F)$, then prove $\dim (W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$. 3(4)

(b) Determine a basis of the subspace spanned by the vectors (3, 2, 4), (1, 0, 2), (1, −1, −1) and (6, 7, 5). 3(4)

Unit II

4\. (a) Let $B = \{u_1, u_2, \ldots\ldots\ldots, u_n\}$ be a basis of a vector space $U(F)$ and $T : U \to U$ be a linear transformation. Then for any vector $u \in U$, prove $[T(u), B] = [T\ B]\ [u, B]$ 2½(4)

(b) Let $u_1 = (1, 1, -1)$, $u_2 = (4, 1, 1)$, $u_3 = (1, -1, 2)$ be a basis of R^3. Let $T : R^3 \to R^2$ be the linear transformation such that $T(u_1) = (1, 0)$, $T(u_2) = (0, 1)$ and $T(u_3) = (1, 1)$. Find T. 2½(4)

5. (a) If $T : U(F) \rightarrow U(F)$ is a linear transformation then Rank T + Nullity T = dim U. 2½(4)

   (b) Find the coefficients of the vector (5, –1, 2) w.r.t. the basis $V_1(1, 4, 2)$, $V_2(4, 2, 1)$, $V_3 = (2, 1, 3)$. 2½(4)

Unit III

6. (a) Find the matrix representing the transformation $T : R^3 \rightarrow R^4$ defined by $T(x, y, z) = (x + y + z, 2x + z, 2y - z, 6y)$ relative to standard basis of R^3 and R^4. 3(4)

   (b) Let $T : R^3 \rightarrow R^2$ be a linear transformation defined by $T(x, y, z) = (2x + y + z, 3x - 2y + 4z)$. Find the matrix T w.r.t. basis $B_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)]$ and $B_2 = \{(1, 3), (1, 4)\}$ of R^3 and R^2 respectively. Also verify $[T : B_1, B_2][u, B_1] = [T(u), B_2]$. 2½(3½)

7. (a) Prove that characteristic polynomial and minimal polynomial of an operator T have the same irreducible factors. 2½(3½)

(b) Prove that minimal polynomial of a matrix of linear operator is unique. 2½(3½)

Unit IV

8. (a) Let V be inner product space, then $|\langle u, v\rangle| \le \|u\|\|v\|$ for all $u, v \in$ V. 2½(3½)

(b) Using Gram Schmidt process, find an orthonormal basis of $V_3(C)$ given the basis $u_1 = (1 + i, i, 1)$, $u_2 = (2, 1 - 2, 2 + i)$, $u_3 = (1 - i, 0, -i)$. 2½(3½)

9. (a) Show that every finite dimensional innerproduct space has an orthonormal basis. 2½(3½)

(b) Show that if α, β are vectors in a unitary space V. then : 2½(3½)

$$\|\alpha+\beta\|^2 - \|\alpha-\beta\|^2 + i\|\alpha+i\beta\|^2 - i\|\alpha-i\beta\|^2 = 4\langle\alpha, \beta\rangle$$