Roll No. ....

(011/17-I)

5161

B.A./B. Sc. EXAMINATION

(First Semester)

MATHEMATICS

BM-113

Solid Geometry

Time : Three Hours *Maxi. Marks :* B.Sc. : 40, B.A. : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Find the nature of the conic : 1½(1)

$$x^2 + 12xy - 4y^2 - 6x + 4y + 9 = 0$$

(b) Find the equation of the normal to the conic : 1½(1)

$$x^2 + 2xy + y^2 - 2x - 1 = 0 \text{ at } (0,\ 1).$$

(c) Find the centre and radius of the sphere : 1½(1)

$$3x^2 + 3y^2 + 3z^2 + 6x - 12z - 12 = 0$$

(d) If a right circular cone has three mutually perpendicular tangent planes, then show that the semi-vertical angle is $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$. 2(1)

(e) Define confocal conicoids. 1½(1)

Section I

2. Trace the conic : 8(5½)

$$8x^2 - 4xy + 5y^2 - 16x - 14y + 17 = 0$$

3. (a). Find the equation of the conic passing through (1, 1) and also through the interdection of the conic $x^2 + 2xy + 5y^2 - 7x - 8y + 6 = 0$ with the straight lines $2x - y - 5 = 0$ and $3x + y - 11 = 0$. 4(3)

(b) Prove that the confocal conics through any point in the plane of ellipse intersect orthogonally. 4(2½)

Section II

4. (a) A variable plane passes through a fixed point (a, b, c) and cuts the co-ordinate axes in the points A, B and C. Show that the locus of the centre of the sphere OABC is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$. 4(3)

(b) Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$ and $2x + 3y + 4z - 8 = 0$ as a great circle. 4(2½)

5. (a) Prove that the equation :

$$7x^2 + 2y^2 + 2z^2 - 10zx + 10xy + 26x - 2y + 2z - 17 = 0$$

represents a cone whose vertex is (1, –2, 2). 4(3)

(b) Find the equation of the right circular cylinder of radius 3 and axis as the line : 4(2½)

$$\frac{x-1}{2} = \frac{y}{2} = \frac{z-3}{1}$$

Section III

6. (a) Find the equation of the tangent planes to the surface $x^2 - 2y^2 + 3z^2 = 2$ which are parallel to the plane $x - 2y + 3z = 0$. 4(3)

(b) The normal at any point P of a central conicoid meets the three principal planes at G_1, G_2, G_3. Show that $PG_1 : PG_2 : PG_3 = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}$. 4(2½)

7. (a) Prove that there are five points on an elliptic paraboloid, the normals at which passes through a given point (α, β, γ). 4(3)

(b) Find the equation of the enveloping cylinder of the conicoid $2x^2 + y^2 + 3z^2 = 1$ whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{2}$. 4(2½)

Section IV

8. (a) If A_1, A_2, A_3 are the areas of three mutually perpendicular central section of an ellipsoid, then show that : 4(3)

$$A_1^{-2} + A_2^{-2} + A_3^{-2} = \text{constant}$$

(b) Find the equation of the generating lines of the hyperboloid $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$, which pass through the point $(2, 3, -4)$ and $\left(2, -1, \frac{4}{3}\right)$. 4(2½)

9. (a) Reduce the equation $2x^2 - 7y^2 + 2z^2 - 10yz - 8zx - 10xy + 6x + 12y - 6z + 5 = 0$ to the standard form and show that it represents a cone. 4(3)

(b) Prove that two conicoids confocal with a given conicoid, touch a given line. 4(2½)