Roll No. .....

(07/20-I)

5261

B. A./B. Sc. EXAMINATION

(Sixth Semester)

MATHEMATICS

BM-363

Dynamics

Time : Three Hours *Max. Marks :* B.Sc. : 40, B.A. : 27

Note : Attempt *Five* questions in all, selecting at least *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) The maximum velocity of a body moving with S.H.M. is 2 unit/sec. and its period is $\frac{1}{5}$ sec. What is its amplitude ? 2(2)

(b) Define Kinetic Energy, Potential Energy, Conservative system of forces and principle of conservation of Energy. 2(1½)

(c) Define horizontal range of a projectile and find the value of maximum horizontal range. 2(1½)

(d) Write down the differential equation of central orbit in Pedal form. 1(1)

(e) Write down the relation between Cartesian co-ordinates and Spherical polar co-ordinates of a point in three dimension. 1(1)

Section I

2. (a) Find the expression for Radial and Transverse components of acceleration of a particle moving along a plane curve $r = f(\theta)$. 4(3)

(b) A particle moves in a straight line with S.H.M. of period 2 sec. If it starts from rest at a distance of 13 cm from the centre of its path, show that the greatest velocity and the velocity acquired by it when it has described 8 cm and 13 π and 12 π cm/sec respectively. 4(2½)

3. (a) Prove that the work done against the tension in stretching a light elastic string is equal to the product of its extension and the mean of the initial and final tensions. 4(3)

(b) A ship steams due west with a velocity of 15 km/hr relative to the current which is flowing at the rate of 6 km/hr due south. What is the velocity of a train going north at a rate of 30 km/hr relative to the ship ? 4(2½)

Section II

4. (a) An engine and train weigh 210 tons and the engine exerts a pull of $3\frac{1}{2}$ tons. The resistance to the motion of train is 14 lbs. wt. per ton. Find the time the train will take to acquire a velocity of 30 m.p.h. from rest. 4(3)

   (b) Prove that the work done in stretching an elastic string is equal to the product of the extension and the mean of initial and final tensions. 4(2½)

5. (a) A particle of mass 'm' falls from rest at a height 'h' above the ground. Show that the sum of kinetic and potential energies is constant throughout the motion. 4(3)

   (b) A train of mass M lbs. is ascending a smooth incline of 1 in n end when the velocity of the train is v ft/sec., its acceleration is f ft/sec^2. Prove that the effective horse power of the engine is $\frac{Mv(nf+g)}{550\,ng}$. 4(2½)

Section III

6. (a) A heavy particle of mass 'm' is made to move on a smooth curve in a vertical plane. Discuss the motion. 4(3)

   (b) A heavy particle slides down a smooth cycloid starting from rest at the cusp, the axis being vertical and vertex downwards. Prove that magnitude of the acceleration is equal to g. 4(2½)

7. (a) A particle of mass 'm' is projected in a vertical plane through the point of projection with velocity 'v' in a direction making an angle α with the horizontal. Find the equation of trajectory and the time of flight. 4(3)

   (b) How must a ball be projected from a height of 4 ft; so as just to clear a wall 13 ft. high, distant 15 ft. in a horizontal direction and a ditch 5 ft. wide on the other side of the wall. 4(2½)

Section IV

8. (a) Derive the differential equation of the central orbit $\frac{d^2u}{d\theta^2}+u=\frac{F}{h^2u^2}$, where the symbols have their usual meanings. 4(3)

(b) A particle describes the equiangular spiral $r=ae^{\theta\cot\alpha}$ under a force to the pole. Find the law of force. 4(2½)

9. (a) Derive the equation of acceleration of a particle moving along a curve :

$$\vec{f}=\frac{dv}{dt}\hat{t}+\frac{v^2}{\rho}\hat{n},$$

where symbols have usual meanings. 4(3)

(b) Deduce the three Kepler's Laws of Planetary Motion from Newton's Law of Gravitation. 4(2½)