

Roll No.

(05/16-I)

5179

B.A./B.Sc. EXAMINATION

(Second Semester)

MATHEMATICS

BM-121

Number Theory and Trigonometry

Time : Three Hours Maximum Marks : $\begin{cases} \text{B.Sc. : 40} \\ \text{B.A. : 27} \end{cases}$

Note : Attempt Five questions in all, selecting *one* question from each Section. Q. No. 1 is compulsory. Marks within the brackets is for B.A. students.

Compulsory Questions

1. (a) If $a|bc$ and $(a, b) = 1$, then prove that
 $a|c$. 2(1½)

- (b) If ϕ is Euler's function, then find $\phi(462)$. $1\frac{1}{2}(1\frac{1}{2})$
- (c) Separate $\sinh(x + iy)$ into real and imaginary parts. $1\frac{1}{2}(1\frac{1}{2})$
- (d) Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$. $1\frac{1}{2}(1\frac{1}{2})$
- (e) Prove that $\exp(2n\pi i) = 1$. $1\frac{1}{2}(1)$

Section I

2. (a) Prove that product of r consecutive positive integers is divisible by $r!$ $4(2\frac{1}{2})$
- (b) Show that there are infinitely many primes of the form $4n+3$. $4(2\frac{1}{2})$
3. (a) The linear congruence $ax \equiv b \pmod{m}$ where a is not congruent to a \pmod{m} has a solution iff (a, m) divides b . $4(2\frac{1}{2})$
- (b) Show that $n^{16} - a^{16}$ is divisible by 85 if n and a are coprime to 85. $4(2\frac{1}{2})$

- Section II**
4. (a) Solve the congruences : $4(2\frac{1}{2})$
 $x \equiv 1 \pmod{4}$
 $x \equiv 3 \pmod{5}$
 $x \equiv 2 \pmod{7}$
- (b) Find the highest power of 15 contained in $736!$ $4(2\frac{1}{2})$
5. (a) Evaluate $\left(-\frac{168}{11}\right)$, where $(\)$ denotes Legendre's symbol. $4(2\frac{1}{2})$
- (b) Prove that Euler's function ϕ is a multiplicative function. $4(2\frac{1}{2})$

Section III

6. (a) Find all the values of $\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{3/4}$ and prove that continued product of all the values is 1. $4(2\frac{1}{2})$

- (b) Express $\cos^6 \theta \sin^4 \theta$ in a series of cosines of multiplex of θ . 4(2½)

7. (a) If $\sin(u+iv) = x+iy$, then prove that :

$$\frac{x^2}{\cosh^2 v} + \frac{y^2}{\sinh^2 v} = 1 \quad \text{4(2½)}$$

- (b) If α, β are imaginary cube roots of unity, then prove that : 4(2½)

$$\alpha \cdot e^{\alpha x} + \beta \cdot e^{\beta x} =$$

$$-e^{-x/2} \left[\sqrt{3} \sin\left(\frac{\sqrt{3}}{2}x\right) + \cos\left(\frac{\sqrt{3}}{2}x\right) \right]$$

Section IV

8. (a) If :

$$\cos(\theta+i\phi) = r(\cos \alpha + i \sin \alpha)$$

$$\text{prove that } \phi = \frac{1}{2} \log \left(\frac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)} \right). \quad \text{4(2½)}$$

- (b) Solve the equation :

$$\tan^{-1}\left(\frac{1}{4}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{6}\right) + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{4}$$

9. (a) If :

$$\cos^{-1}(\alpha + i\beta) = \theta + i\phi$$

$$\text{prove that } \alpha^2 \operatorname{sech}^2 \phi + \beta^2 \cosh^2 \phi = 1. \quad \text{4(2½)}$$

- (b) Sum to n terms the series :

$$\cot^{-1}(2 \cdot 1^2) + \cot^{-1}(2 \cdot 2^2) + \cot^{-1}(2 \cdot 3^2) + \dots \quad \text{4(2½)}$$