Roll No. ....

(06/21-II)

5260

B.A./B.Sc. EXAMINATION

(For Batch 2011 & Onwards)

(Sixth Semester)

MATHEMATICS

BM-362

Linear Algebra

Time : Three Hours *Maximum Marks* : B.Sc. : 40, B.A. : 26

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. Marks outside the brackets is for B.Sc. Students and marks within the brackets is for B.A. Students.

(Compulsory Question)

1. (a) Define linear sum and direct sum of two subspaces of a vector space. 2(1½)

(b) Define basis of a vector space. 2(1½)

(c) Let $T : U \to V$ be a linear transformation. Then prove that R(T) i.e., range of T is a subspace of V. 2(1½)

(d) Prove that $\|au\| = |a|\|u\|$, for all $a \in F$, $u \in V$, where V is an inner product space. 2(1½)

Unit I

2. (a) Prove that a non-empty subset W of a vector space V(F) is a subspace of V if and only if $au + v \in W$ for each $a \in F$ and $u, v \in W$. 4(2½)

(b) Prove that necessary and sufficient conditions for a vector space V(F) to be direct sum of its subspaces W_1 and W_2 are that : 4(2½)

(i) $V = W_1 + W_2$

(ii) $W_1 \cap W_2 = \{0\}$

3. (a) Prove that the set $\{(2, 1, 4), (1, -1, 2), (3, 1, -2)\}$ forms a basis of R^3. 4(2½)

(b) Let W be a subspace of a finite dimensional vector space V(F), then dim.(V/W) = dim. V − dim. W. 4(2½)

Unit II

4. (a) Two finite dimensional vector spaces over the same field are isomorphic if and only if they have the same dimension. 4(2½)

(b) Find the linear transformation which maps (1, 1, 1), (1, 1, 0), (1, 0, 0) in R^3 to (2, 1), (2, 1), (2, 1) in R^2. 4(2½)

5. (a) If $T : R^4 \to R^3$ is a linear transformation defined by $T(e_1) = (1, 1, 1)$, $T(e_2) = (1, -1, 1)$, $T(e_3) = (1, 0, 0)$, $T(e_4) = (1, 0, 1)$, then verify that : 4(2½)

$$\rho(T) + \mu(T) = \dim R^4 = 4$$

(b) Prove that the transformation $T : R^3 \rightarrow R^2$ defined by $T(x_1, x_2, x_3) = (x_1, x_2)$ is a linear transformation and is onto but not one-one. 4(2½)

Unit III

6. (a) If $T : R^3 \rightarrow R^3$ be a linear operator defined by $T(x, y, z) = (x + z, x - z, y)$. Show that T is invertible and find T^{-1}. 4(2½)

(b) Let $T : R^3(R) \rightarrow R^3(R)$ be a linear transformation defined by $T(x, y, z) = (2x - y, x + y + z, 2z)$. Find the characteristic and minimal polynomial for T. 4(2½)

7. Let $T : R^3 \rightarrow R^2$ be a linear transformation defined by $T(x, y, z) = (2x + y - z; 3x - 2y + 4z)$. Find the matrix of T with respect to ordered basis $B_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and $B_2 = \{(1, 3), (1, 4)\}$ of R^3 and R^2 respectively. Also verify that : 8(5)

$$[T : B_1, B_2][u, B_1] = [T(u), B_2]$$

Unit IV

8. (a) State and prove Bassel's inequality. 4(2½)

(b) Let V be an inner product space. If u, $v \in V$ such that $|\langle u, v\rangle| = \|u\|\|v\|$, then show that u and v are linearly dependent. 4(2½)

9. (a) State and prove Gram-Schmidt orthogonalization process. 4(2½)

(b) Let T be a linear operator on an inner product space V(F). Then T* exists and $TT^* = T^*T = I$ iff T is unitary. 4(2½)