

(b) Find the Mobius transformation which maps the points  $z_1 = 2$ ,  $z_2 = i$ ,  $z_3 = -2$  into the points  $w_1 = 1$ ,  $w_2 = i$ ,  $w_3 = -1$ .

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Roll No. ....

(06/21-II)

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### B. A/B. Sc. EXAMINATION

(For Batch 2011 & Onwards)

(Sixth Semester)

MATHEMATICS

BM-361

Real & Complex Analysis

Time : Three Hours Maximum Marks :  $\begin{cases} \text{B.Sc. : 40} \\ \text{B.A. : 27} \end{cases}$

**Note :** Attempt Five questions in all, selecting one question from each Unit and the compulsory question. Marks in brackets are for B.A. students.

#### (Compulsory Question)

1. (a) Show that  $u = \sin x + \sin y$ ,  $v = \sin(x + y)$  are not functionally dependent.  $1\frac{1}{2}(1\frac{1}{2})$

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- (b) Define Dirichlet's conditions for Fourier expansion.  $1\frac{1}{2}(1\frac{1}{2})$
- (c) Define differentiability of a complex function.  $1\frac{1}{2}(1)$
- (d) Determine the image of the following points on the sphere of radius  $\frac{1}{2}$  and centre  $(0, 0, \frac{1}{2})$  :  $1\frac{1}{2}(1)$
- (i)  $1 + i$
- (ii)  $1 - i$
- (iii)  $2 + 3i$
- (e) Define conformal mapping.  $1(1)$
- (f) Define the following terms :  $1(1)$
- (i) Magnification
- (ii) Rotation.

### Unit I

2. (a) Show that the functions  $u = x^2 + y^2 + z^2$ , and  $v = xy - xz - yz$ ,  $w = x + y - z$  are dependent. Also find the relation connecting them.  $4(2\frac{1}{2})$

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- (b) Prove that :

$$B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

where  $m > 0, n > 0$ .

$4(2\frac{1}{2})$

3. (a) Evaluate :

$4(2\frac{1}{2})$

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz \, dz \, dy \, dx$$

- (b) Evaluate :

$$\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$$

Change the order of integration and verify the result.  $4(2\frac{1}{2})$

### Unit II

4. (a) Obtain the Fourier expansion for  $f(x)$ ,  
if  $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$   $4(2\frac{1}{2})$
- (b) Express  $f(x) = x$  as a half range sine series in  $0 < x < 2$ .  $4(2\frac{1}{2})$

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5. (a) Given the series  $x = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$ ,  
show that :

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

using Parseval's identity.

4(2½)

- (b) Obtain the Fourier expansion for  $f(x)$ ,  
if  $f(x) = x \sin x$  in  $(0, 2\pi)$ .

4(2½)

### Unit III

6. (a) If the function :

$$f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z - i}$$

continuous at  $z = i$  ? If not, give reason.

4(2½)

- (b) Construct the analytic function of which  
real part is :

4(2½)

$$u(x, y) = e^x (x \cos y - y \sin y)$$

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7. (a) Prove that an analytic function with  
constant modulus is constant.

4(2½)

- (b) Show that the function

$$f(z) = e^{-z^{-4}} \quad (z \neq 0) \text{ and } f(0) = 0 \text{ is not}$$

analytic at  $z = 0$ , although C-R equations  
are satisfied at that point.

4(2½)

### Unit IV

8. (a) Let the rectangular region  $D$  in the  
 $z$ -plane be bounded by  $x = 0$ ,  $y = 0$ ,  
 $x = 1$ ,  $y = 2$ . Determine the region  $D'$  of  
the  $w$ -plane into which  $D$  is mapped  
under the transformation  $w = \sqrt{2} e^{i\frac{\pi}{4}} z$ .

4(2½)

- (b) Prove that every Mobius transformation  
is the resultant of Mobius transformation  
with simple geometric imports.

4(2½)

9. (a) Define cross-ratio and show that cross-  
ratio remains invariant under Mobius  
transformation.

4(2½)

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