

Roll No.

(06/21-II)

5220

B.A./B.A.(Hons.)/B.Sc. EXAMINATION

(For Batch 2011 & Onwards)

(Fourth Semester)

MATHEMATICS

BM-242

Special Foundations and Integral Transforms

Time : Three Hours Maximum Marks : $\begin{cases} \text{B.A. : 26} \\ \text{B.Sc. : 40} \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory.

(Compulsory Question)

1. (a) Show that :

$$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{\sin x}{x} - \cos x \right]$$

- (b) Using Rodrigue's formula, find Legendre Polynomial $P_2(x)$.
2(1)

- (c) Find Laplace transform of $\cos^3(2t)$.
2(1)

- (d) Find :
1(1)

$$L^{-1} \left[\frac{1}{s} \sin \frac{1}{s} \right]$$

- (e) Find Fourier sine transform of function

$$f(x) = 2e^{-5x} + 5e^{-2x}.$$

1(1)

Section I

2. (a) Show that $x = 0$ is an ordinary point of

$$\left(x^2 - 1 \right) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0, \text{ but } x = 1 \text{ is}$$

a regular singular point.
4(2½)

- (b) Find Power series solution of differential equation :
4(2½)

$$\left(1 + x^2 \right) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0$$

3. (a) Show that :

$$J_{-n}(x) = (-1)^n J_n(x)$$

where n is any integer.
4(2½)

- (b) Use Jacobi series to show that :
4(2½)

$$[J_0(x)]^2 + 2[J_1(x)]^2 + 2[J_2(x)]^2 + \dots = 1$$

Section II

4. (a) Using generating function for Legendre Polynomials show that :
4(2½)

$$P'_{2n}(0) = 0$$

- (b) Prove that :
4(2½)

$$\int_{-1}^1 x P_n(x) P'_n(x) dx = \frac{2n}{2n+1}$$

4(2½)

5. (a) Show that :
 $H'_n(x) = 2nH_{n-1}(x)$, $x \geq 1$
4(2½)

- (b) Evaluate :
4(2½)

$$\int_{-\infty}^{\infty} x e^{-x^2} H_n(x) H_m(x) dx$$

Section III

6. (a) Find Laplace Transform of $t^2 \cdot \cos at$.

4(2½)

- (b) Evaluate :

$$\int_0^\infty t^3 \cdot e^{-t} \cdot \sin t \, dt$$

7. (a) Apply convolution theorem to evaluate :

4(2½)

$$L^{-1} \left[\frac{s}{(s^2 + a^2)^2} \right]$$

- (b) Solve the following equation by Laplace Transform :

4(2½)

$$\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 3y = e^{-x}, y(0) = y'(0) = 1$$

Section IV

8. (a) Find Fourier cosine transform of e^{-x^2} .

4(2½)

- (b) Using Parseval Identity, prove that : 4(2½)

$$\int_0^\infty \frac{dx}{(x^2 + 1)^2} = \frac{\pi}{4}$$

9. (a) Find the finite sine transform of :

$$f(x) = 2x$$

where $0 < x < 4$.

4(2½)

- (b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, given that :

- (i) $u(0, t) = 0$
- (ii) $u(\pi, t) = 0$
- (iii) $u(x, 0) = 2x$.