Roll No. ....

(011/17-I)

5159

B. A./B. Sc. EXAMINATION

(First Semester)

MATHEMATICS : ALGEBRA

BM-111

Time : Three Hours *Maxi. Marks* : $\begin{cases} \text{B.Sc.} : 40 \\ \text{B.A.} : 27 \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory. Marks within Brackets is for B.A. Students.

(Compulsory Question)

1. (a) If A is a square matrix; prove that $A-\mathring{A}$ is skew-Hermitian. 2(1)

   (b) Prove that determinant of an orthogonal matrix is ± 1. 2(1)

(c) Find an equation whose roots are four times and with their sign changed of the roots of the equation $4x^4 - x^3 + x + 1 = 0$. 2(1½)

(d) Show that the equation $2x^7 - 5x^4 + 3x^3 - 1 = 0$ has at least four imaginary roots. 2(1½)

Section I

2. (a) Show that every square matrix can be expressed as the sum of a symmetric and skew-symmetric matrix in one and only one way. 4(2½)

(b) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 3 \\ -2 & -4 & 4 & -7 \\ 1 & 2 & 1 & 2 \end{bmatrix}$ by reducing it to normal form. 4(3)

3. (a) Determine the characteristics roots and corresponding characteristic vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. 4(2½)

(b) Prove that characteristics roots of a Hermitian matrix are all real. 4(3)

Section II

4. (a) Solve : 4(2½)

$$x + y + 2z + w = 5$$

$$2x + 3y - z - 2w = 2$$

$$4x + 5y + 3z = 7$$

(b) Show that the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is orthogonal iff $A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ or $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$, where $a^2 + b^2 = 1$. 4(3)

5. Reduce the bilinear form $x_1y_1 + x_1y_3 - x_2y_1 + x_2y_2 + x_3y_3$ to the canonical form. Also find the equations of transformation. 8(5½)

Section III

6. (a) Solve the equation $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$, roots being in A.P. 4(2½)

(b) Find the common roots of the equations $x^4 + 3x^3 - 5x^2 - 6x - 8 = 0$ and $x^4 + x^3 - 9x^2 + 10x - 8 = 0$. Hence solve them completely. 4(3)

7. (a) Form the equation whose roots are cubes of the roots of the equation $x^3 + px^2 + qx + r = 0$. 4(2½)

(b) Find the equation of squared differences of the roots of the equation $x^3 - 7x + 6 = 0$. 4(3)

51

Section IV

8. (a) Discuss the nature of the roots of the cubic $Z^3 + 3HZ + G = 0$. 4(2½)

   (b) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method. 4(3)

9. (a) Solve the equation $x^4 - 4x^3 + 9x^2 - 12x + 18 = 0$ by Descarte's method. 4(2½)

   (b) Solve the equation $x^4 + 4x^3 + 12x^2 - 8x + 95 = 0$ by Ferrari's method. 4(3)